Sample statistic	Population statistic	Sample statistic formula	Population statistic formula
$\overline{X}$	μ	$\overline{X} = \frac{1}{n}\sum X$	$\mu = E(X)$ which is $= \sum X \cdot P(X)$ for discrete distributions
s	σ	$s = \sqrt{\frac{1}{n-1}\sum (X-\overline{X})^2}$	$\sigma = \sqrt{E\left((X-\mu)^2\right)}$ which is $= \sqrt{\sum (X-\mu)^2 \cdot P(X)}$ for discrete distributions
$\hat{p}$	p	$\hat{p} = \frac{X}{n}$ where X is a binomial random variable measuring the number of successes in a sample of size, n.	$p = \frac{X}{N}$ where X is a binomial random variable measuring the number of successes in the population of size, N.
$\overline{p}$	$p = p_1 = p_2$ under the null hypothesis	$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2}$	$p_1 = \frac{X_1}{N_1}$ and $p_2 = \frac{X_2}{N_2}$

The sample statistics, above, estimate the corresponding population statistics. They are point estimators. To do an interval estimate (confidence interval) or a hypothesis test on a population statistic, it is necessary to understand a related probability distribution.

If X is a random variable with a normal distribution of mean, μ, and standard deviation, σ, then $Z = \frac{X-\mu}{\sigma}$ also has a normal distribution; but now the mean is 0 and the standard deviation is 1.

It should also make sense that $X - \overline{X}$ has a mean of zero and is actually still normally distributed. So $Z = \frac{X-\overline{X}}{\sigma}$ is also normal, with mean, 0, and standard deviation, 1.

It turns out that the sum of the squares of n standard normal random variables has a chi-squared distribution with $n-1$ degrees of freedom. So, $\chi^2 = \sum \frac{(X-\overline{X})^2}{\sigma^2}$ has a chi-squared distribution with $n-1$ degrees of freedom. But from the formula for s, above, you can see that $\chi^2 = \sum \frac{(X-\overline{X})^2}{\sigma^2} = \frac{(n-1)s^2}{\sigma^2}$. This is how confidence intervals and hypothesis tests for σ and σ^2 are justified.

For the binomial random variable X, a normal random variable with $\mu = np$ and $\sigma = \sqrt{npq}$ will approximate it. This means that: $Z = \frac{X-\mu}{\sigma} = \frac{X-np}{\sqrt{npq}} = \frac{\frac{X}{n} - p}{\frac{\sqrt{npq}}{n}} = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$ (where we divided the numerator and denominator by n), giving the familiar result.

A *t*-distribution is obtained from the quotient of a standard normal random variable, *Z*, divided by the square root of a χ^2 random variable which has been divided by it's degrees of freedom. The result has a *t*-distribution with the same degrees of freedom as the χ^2 random variable in the denominator: $t = \dfrac{Z}{\sqrt{\dfrac{\chi^2}{n-1}}}$.

This means that $t = \dfrac{Z}{\sqrt{\dfrac{\chi^2}{n-1}}} = \dfrac{\dfrac{\overline{X}-\mu}{\dfrac{\sigma}{\sqrt{n}}}}{\sqrt{\dfrac{(n-1)\dfrac{s^2}{\sigma^2}}{n-1}}} = \dfrac{\dfrac{\overline{X}-\mu}{\dfrac{\sigma}{\sqrt{n}}}}{\sqrt{\dfrac{s^2}{\sigma^2}}} = \dfrac{\overline{X}-\mu}{\dfrac{\sigma}{\sqrt{n}}} \cdot \dfrac{\sigma}{s} = \dfrac{\overline{X}-\mu}{\dfrac{s}{\sqrt{n}}}$ since $\dfrac{\sigma}{\sqrt{n}}$ is the standard deviation of $\overline{X}$ and $\dfrac{(n-1)s^2}{\sigma^2} = \chi^2$. This gives us the familiar *t* test statistic: $t = \dfrac{\overline{X}-\mu}{\dfrac{s}{\sqrt{n}}}$.

An *F* distribution is obtained from the quotient of two chi-squared random variables, each of which has been divided by its degrees of freedom. Under the null hypothesis that $\sigma_1 = \sigma_2$, this means that if $\chi_1^2 = \dfrac{(n_1-1)s_1^2}{\sigma_1^2}$ and $\chi_2^2 = \dfrac{(n_2-1)s_2^2}{\sigma_2^2}$ then $\dfrac{\dfrac{\chi_1^2}{n_1-1}}{\dfrac{\chi_2^2}{n_2-1}} = \dfrac{\dfrac{s_1^2}{\sigma_1^2}}{\dfrac{s_2^2}{\sigma_2^2}} = \dfrac{s_1^2}{s_2^2}$ will have an *F* distribution with d.f.N. $= n_1 - 1$ and d.f.D. $= n_2 - 1$; (degrees of freedom in the numerator and denominator, respectively).